Answer Four questions only

(1)(a) Using the mathematical induction, prove that: $6 + 8 + 10 + \ldots + (2n + 4) = n(n + 5)$.

(b) Using the binomial theorem, expand $\dfrac{1}{\sqrt{9 - 3x}}$.

(c) Find the sum $\displaystyle\sum_{r=1}^{n} \dfrac{1}{(r + 3)(r + 4)}$.

(2)(a) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \end{bmatrix}$. Find, if possible, $A + B$, $|A|$, $|B|$, AB and BA.

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$.

(c) Solve the linear system $x + y - z = 2$, $2x + 3y - 3z = 3$, $3x + 4y - 4z = 5$.

(3)(a) Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$ where its roots form arithmetic progression.

(b) Identify the following surfaces:

(i) $x^2 + 4y^2 + 8z = 0$

(ii) $x^2 - 2y^2 - 4z^2 - 4x + 12y + 32z = 0$

(4)(a) Find the angle of rotation of axes such that the term containing xy in the equation

$5x^2 + 2xy + 5y^2 = 2$ disappears.

(b) Find the equation of the line pair through the origin perpendicular to the line pair

$x^2 + 3xy + 2y^2 - x + y - 6 = 0$.

Find also the angle between them and the equation of the bisectors.

(5) Describe the following curves: (a) $16x^2 + 9y^2 - 32x + 36y - 92 = 0$

(b) $3x^2 = -8y$ (c) $16x^2 - 49y^2 = 784$

Good luck   *Dr. Mona Mehanna*

Dr. Mohamed Husein Eid

(1)Using the mathematical induction, prove that: $\dfrac{1}{4\times5}+\dfrac{1}{5\times6}+...+\dfrac{1}{(n+3)(n+4)}=\dfrac{n}{4(n+4)}$

(2) Using the binomial theorem, expand $\dfrac{1}{2x+3}$.

(3) Find the sum $\displaystyle\sum_{r=1}^{n}\dfrac{2}{(2r+1)(2r+3)}$.

(4) If A$=\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$ and B $=\begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix}$.

Find, if possible, A + B, |A|, BA, AB, A^{-1}, B^{-1}

QuizI: Name: _____ .

(1)Using the mathematical induction, prove that

$$\frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \ldots + \frac{1}{(n+3)(n+4)} = \frac{n}{4(n+4)}$$

(2)Find the nth sum $\displaystyle\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)}$

(3)Using the binomial expansion, expand $\dfrac{1}{\sqrt{4-2x}}$

Quiz2

(1)Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$

(2) Show that the matrix $B = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal.